The Row Matrix

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Background and examples

As is well known, all the rows in a classical row-class (consisting of all transpositions of rows P, RP, IP, and RIP) can be concisely listed using a 12 by 12 square of pitch-class numbers called a row matrix.[1] The row matrix appeared in the 1950s (probably invented by Milton Babbitt); thus, the members of the second Viennese school and other early twelve-tone composers did not use it; they usually listed the 48 rows written as pitches on the staff.

The following is a row matrix for row S = <024761A5B398>

0	2	4	7	6	1	A	5	B	3	9	8
A	0	2	5	4	B	8	3	9	1	7	6
8	A	0	3	2	9	6	1	7	B	5	4
5	7	9	0	B	6	3	A	4	8	2	1
6	8	A	1	0	7	4	B	5	9	3	2
B	1	3	6	5	0	9	4	A	2	8	7
2	4	6	9	8	3	0	7	1	5	B	A
7	9	B	2	1	8	5	0	6	A	4	3
1	3	5	8	7	2	B	6	0	4	A	9
9	B	1	4	3	A	7	2	8	0	6	5
3	5	7	A	9	4	1	8	2	6	0	B
4	6	8	B	A	5	2	9	3	7	1	0

The rows on the row matrix of S are listed as follows:

The T_nS rows are listed on matrix rows from left to right.
The RT_nS rows are listed on matrix rows from right to left.
The T_nIS rows are listed on matrix columns from top to bottom.
The RT_nIS rows are listed on matrix columns from bottom to top.

[1] The row matrix has been also called the row table, set table, Babbitt square, and magic square.

For example,

The row S <024761A5B398> is listed from left to right on the top matrix row.

The row RT_7S <34A605812B97> is listed from right to left on the 8th matrix row.

The row T_4IS <4209A36B5178> is listed from top to bottom on the third column of the matrix

The row RT_3IS <7604A5298B13> is listed from bottom to top on the 10th column of the matrix.

Here is the matrix showing these rows in bold face.

0	**2**	**4**	**7**	**6**	**1**	**A**	**5**	**B**	**3**	**9**	**8**
A	0	**2**	5	4	B	8	3	9	**1**	7	6
8	A	**0**	3	2	9	6	1	7	**B**	5	4
5	7	**9**	0	B	6	3	A	4	**8**	2	1
6	8	**A**	1	0	7	4	B	5	**9**	3	2
B	1	**3**	6	5	0	9	4	A	**2**	8	7
2	4	**6**	9	8	3	0	7	1	**5**	B	A
7	**9**	**B**	**2**	**1**	**8**	**5**	**0**	**6**	**A**	**4**	**3**
1	3	**5**	8	7	2	B	6	0	**4**	A	9
9	B	**1**	4	3	A	7	2	8	**0**	6	5
3	5	**7**	A	9	4	1	8	2	**6**	0	B
4	6	**8**	B	A	5	2	9	3	**7**	1	0

Note that the T_nS rows are not listed “in order” from top to bottom with n advancing from 0, to 1, to 2, etc. to B. Rather, they are listed from top to bottom such that their first notes form the notes of the IS row.

Likewise, the T_nIS rows are listed from left to right so that their first notes form the notes of the S row.

Constructing a row matrix

The row matrix is constructed from a row P and the inversion of P that starts with first note of P.[2] For convenience, row tables are constructed for rows that begin with pc 0.[3] The P row is written on the top row of the matrix; the IP row is written as the leftmost column of the matrix as shown immediately below. P is <0164A9B83752>.

0	1	6	4	A	9	B	8	3	7	5	2
B											
6											
8											
2											
3											
1											
4											
9											
5											
7											
A											

The rest of the rows are added. A T_nP row is written on the matrix row that has n on the left. A T_nIP row is written on the matrix column that has n on the top row. Here is partially completed row matrix that has P, T_nP and T_nIP rows where n is 0, 1, 2, and 3.

0	1	6	4	A	9	B	8	3	7	5	2
B	0							2			1
6	7							9			8
8	9							B			A
2	3	8	6	0	B	1	A	5	9	7	4
3	4	9	7	1	0	2	B	6	A	8	5
1	2	7	5	B	A	0	9	4	8	6	3
4	5							7			6
9	A							0			B
5	6							8			7
7	8							A			9
A	B							1			0

[2] If P does not start with 0 or 6, then IP will not start with 0 or 6.

[3] If a row does not start on 0, one can transpose it so it does and label the transposition as P, or redefine the pcs so pc 0 is the first note of the row.

Here is the completed row matrix.

0	1	6	4	A	9	B	8	3	7	5	2
B	0	5	3	9	8	A	7	2	6	4	1
6	7	0	A	4	3	5	2	9	1	B	8
8	9	2	0	6	5	7	4	B	3	1	A
2	3	8	6	0	B	1	A	5	9	7	4
3	4	9	7	1	0	2	B	6	A	8	5
1	2	7	5	B	A	0	9	4	8	6	3
4	5	A	8	2	1	3	0	7	B	9	6
9	A	3	1	7	6	8	5	0	4	2	B
5	6	B	9	3	2	4	1	8	0	A	7
7	8	1	B	5	4	6	3	A	2	0	9
A	B	4	2	8	7	9	6	1	5	3	0

Properties of the row matrix

1. As mentioned above, the P rows are listed in the row matrix in the order of the pcs in the IP row, and vice versa.

2. The main diagonal—the diagonal from the upper left to the lower right—contains only pc 0.

3. Any two pcs symmetrically disposed around the main diagonal are related by I. Thus, for pc a in the nth row and the mth column, we have pc Ia in the mth row and nth column. For example, in the row matrix: 3 is in the 2nd matrix row and 4th column and 9 (= I3) is in the 4th matrix row and 2nd column; 8 is in the 8th matrix row and 4th column and 4 (= I8) is in the 4th matrix row and 8th column.

4a. The successive intervals of P called the *INT(P)*[4] occur in the diagonal to the immediate right of the main diagonal.

The INT(P) = <15A6B2974A9> and it found on the matrix in bold face as shown on the next page.

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[4] For information on the INT and INT_n function on rows, see my Elementary Twelve-Tone Theory document.

0	**1**	6	4	A	9	B	8	3	7	5	2
B	0	**5**	3	9	8	A	7	2	6	4	1
6	7	0	**A**	4	3	5	2	9	1	B	8
8	9	2	0	**6**	5	7	4	B	3	1	A
2	3	8	6	0	**B**	1	A	5	9	7	4
3	4	9	7	1	0	**2**	B	6	A	8	5
1	2	7	5	B	A	0	**9**	4	8	6	3
4	5	A	8	2	1	3	0	**7**	B	9	6
9	A	3	1	7	6	8	5	0	**4**	2	B
5	6	B	9	3	2	4	1	8	0	**A**	7
7	8	1	B	5	4	6	3	A	2	0	**9**
A	B	4	2	8	7	9	6	1	5	3	0

4b. The $INT_n(P)$ occurs in the diagonal n diagonals to the right of the main diagonal.

The $INT_2(P)$ is given in bold face; the $INT_4(P)$ is given underlined

0	1	**6**	4	A	9	B	8	3	7	5	2
B	0	5	**3**	9	8	A	7	2	6	4	1
6	7	0	A	**4**	3	5	2	9	1	B	8
8	9	2	0	6	**5**	7	4	B	3	1	A
2	3	8	6	0	B	**1**	A	5	9	7	4
3	4	9	7	1	0	2	**B**	6	A	8	5
1	2	7	5	B	A	0	9	**4**	8	6	3
4	5	A	8	2	1	3	0	7	**B**	9	6
9	A	3	1	7	6	8	5	0	4	**2**	B
5	6	B	9	3	2	4	1	8	0	A	**7**
7	8	1	B	5	4	6	3	A	2	0	9
A	B	4	2	8	7	9	6	1	5	3	0

4c. The INT(IP) occurs in the diagonal to the immediate left of the main diagonal.

4d. The $INT_n(IP)$ occurs in the diagonal n diagonals to the left of the main diagonal.

The INT(IP) is shown in boldface and the $INT_3(IP)$ is shown underlined on the matrix on the next page.

0	1	6	4	A	9	B	8	3	7	5	2
B	0	5	3	9	8	A	7	2	6	4	1
6	**7**	0	A	4	3	5	2	9	1	B	8
8	9	**2**	0	6	5	7	4	B	3	1	A
2	3	8	**6**	0	B	1	A	5	9	7	4
3	4	9	7	**1**	0	2	B	6	A	8	5
1	2	7	5	B	**A**	0	9	4	8	6	3
4	5	A	8	2	1	**3**	0	7	B	9	6
9	A	3	1	7	6	8	**5**	0	4	2	B
5	6	B	9	3	2	4	1	**8**	0	A	7
7	8	1	B	5	4	6	3	A	**2**	0	9
A	B	4	2	8	7	9	6	1	5	**3**	0

5a. The pcs on the secondary diagonal (the diagonal from the lower left to the upper right) forms a sequence of intervals derived from P in order as follows:

the interval from the last pc of P to the first pc of P;
the interval from the next to last pc of to the second pc of P;
the interval from the third from last pc of P to the third pc of P;
etc.;
the interval from the second pc of P to the next to last pc of P;
the interval from the first pc of P to the last pc of P.

In other words, the sequence of entries on the secondary diagonal is the sequence of intervals from $P_{B\text{-}n}$ to P_n as n advances from 0 to 11 (and n is taken mod-12).

Let us construct the series from P = <0164A9B83752>

First we take the interval from the last pc of P to the first; this is the interval from 2 to 0; it is A.
Second, we take the interval from the next to last pc of P to the second pc; it is 8, the interval from 5 to 1.

The series so far is < A 8 ...

The interval from the tenth pc of P (P_9) to the third pc (P_2) is B, the interval from 7 to 6.
The interval from P_8 = 3 to P_3 = 4 is 1.

The series is now <A 8 B 1 ...

... etc...

The interval from P_1 to P_A is 4.
The interval from P_0 to P_B is 2.

The completed series is <A8B12A2AB142>. Note that this kind of series is always invariant under retrograde inversion.

The series is shown given on the secondary diagonal of the row matrix in bold face.

0	1	6	4	A	9	B	8	3	7	5	**2**
B	0	5	3	9	8	A	7	2	6	**4**	1
6	7	0	A	4	3	5	2	9	**1**	B	8
8	9	2	0	6	5	7	4	**B**	3	1	A
2	3	8	6	0	B	1	**A**	5	9	7	4
3	4	9	7	1	0	**2**	B	6	A	8	5
1	2	7	5	B	**A**	0	9	4	8	6	3
4	5	A	8	**2**	1	3	0	7	B	9	6
9	A	3	**1**	7	6	8	5	0	4	2	B
5	6	**B**	9	3	2	4	1	8	0	A	7
7	**8**	1	B	5	4	6	3	A	2	0	9
A	B	4	2	8	7	9	6	1	5	3	0

These row matrix properties and others will be explained and elaborated when we show that the row matrix is an instance of a structure called an *invariance matrix*.